

Models of mixed hadron-quark-gluon matter

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Abstract

The problem of the possible creation of mixed hadron-quark-gluon matter, that can arise at nuclear or heavy-ion collisions, is addressed. It is shown that there can exist several different kinds of such a mixed matter. The main types of this matter can be classified onto macroscopic mixture, mesoscopic mixture, and microscopic mixture. Different types of these mixtures require principally different descriptions. Before comparing theoretical results with experiments, one has to analyze thermodynamic stability of all these mixed states, classifying them onto unstable, metastable, and stable. Only the most stable mixed state should be compared with experiment. Mixed states also need to be checked with regard to stratification instability. In addition to the static stratification instability, there can happen dynamic instability occurring in a mixture of components moving with respect to each other. This effect, called counterflow instability, has also to be taken into account, since it can lead to the stratification of mixed matter.

1 Introduction

At high temperatures and/or densities, hadronic matter is expected to undergo a transition to quark-gluon plasma, where quarks and gluons are no longer confined inside hadrons but can propagate much further in extent than the typical sizes of hadrons. Such a deconfinement transition can happen under heavy-ion or nuclear collisions. It is assumed to exist in the early universe cosmology, since for a time on the order of the microsecond the temperature was high enough for the elementary degrees of freedom of QCD to be in a deconfined state. The quark-gluon plasma can also exist in the interior of compact stars.

The peculiarities of the transition from hadronic matter to quark-gluon plasma, that is, of the deconfinement transition, have been the object of many discussions (see review articles [1-10]). From general arguments, it is impossible to infer the order of the QCD transition, whether it is 1-st order, 2-nd order, or crossover. Being based on a model consideration, the deconfinement was shown to be a gradual crossover [6,7]. At the present time, this result has been confirmed by numerical simulations of lattice QCD showing convincingly that deconfinement is really a crossover [11-15].

In order to be able to describe the states of matter and phase transitions in thermodynamic terms, it is required that the matter be at least in quasi-equilibrium. The experimental lifetime of fireballs, formed under heavy-ion collisions, is of order $t_{exp} \sim 10^{-22}$ s [16,17]. The local equilibration time of nuclear matter is $t_{loc} \sim 10^{-23}$ s [18,19]. Since $t_{loc} \ll t_{exp}$, equilibration is feasible and thermodynamic language is applicable to treating the fireball states.

A plausible assumption is that in the process of the transformation of hadronic matter into quark-gluon plasma there can arise an intermediate state of matter representing a mixture of hadronic and quark-gluon states [20,21]. Note that the manifestation of quark degrees of freedom, resulting in the appearance of the Blokhintsev fluctons [22], Baldin cumulative effect [23], and in the formation of multi-quark clusters, has also been assumed to occur even at temperatures essentially lower than the deconfinement point [24-31].

However, the nature of the mixed hadron-quark-gluon state has not been well understood. It is the aim of the present paper to explain that, actually, there can exist several kinds of such a mixed state, with the main three types that can be classified onto macroscopic, mesoscopic, and microscopic mixed states. These states have rather different properties and require essentially different theoretical description.

In the paper, we use the system of units, where the Planck and Boltzmann constants are set to one.

2 Macroscopic mixed state

This type of mixed state would arise if the deconfinement transition would be of first order [32-34]. Then, at the phase-transition point, the system, say fireball of a linear size L , separates into macroscopic domains of size l corresponding to hadron phase and quark-gluon phase, so that

$$a \ll l \sim L, \quad (1)$$

where a is mean interparticle distance. The domains are macroscopic, being of order of the system size L . They also are called droplets or blobs, or bubbles [35-40]. Their

topology is similar to the droplets of nucleons arising in the low-density nuclear matter [41,42]. The domains of different phases correspond to different vacua [43-46], with the physical states of different domains being mutually orthogonal [47].

Strictly speaking, under first-order phase transition, mixed phase can occur only at the transition point, where two kinds of pure phases meet each other, pure hadron phase and pure quark-gluon phase. The qualitative behavior of the transition temperature T_c as a function of baryon density ρ_B is shown in Fig. 1. Hadron matter consists of only hadrons, interacting with each other through hadron-hadron interactions [48,49]. Pure quark-gluon plasma is described by an equation of state for free quarks and gluons, with taking into account their interactions [50] by incorporating some non-perturbative effects [51-53].

The straightforward order parameters are the density of hadron matter, ρ_h and the density of quark-gluon plasma, ρ_q . In the hadronic matter

$$\rho_h > 0, \quad \rho_q \equiv 0, \quad (2)$$

while in the quark-gluon plasma

$$\rho_h \equiv 0, \quad \rho_q > 0. \quad (3)$$

It is also possible to use as an order parameter the Wilson loop [54,55].

Each type of particles is characterized by baryon number B_i and strangeness S_i . For simplicity, the particles are assumed to be neutral. The chemical potential of the i -type particles is expressed as

$$\mu_i = \mu_B B_i + \mu_S S_i \quad (4)$$

through the baryon, μ_B , and strangeness, μ_S chemical potentials. The baryon and strangeness densities are given by the relations

$$\rho_B = \sum_i B_i \rho_i = \frac{\partial P}{\partial \mu_B}, \quad \rho_S = \sum_i S_i \rho_i = \frac{\partial P}{\partial \mu_S}, \quad (5)$$

in which P is pressure and ρ_i is the density of the i -type particles. The behavior of pressure, under first-order phase transition, is shown in Fig. 2. Since the grand potential is $\Omega = -PV$, where V is the system volume, the larger pressure corresponds to the lower grand potential.

The transition temperature is defined by the equality of the pressures,

$$P_h(T_c, \mu_B) = P_q(T_c, \mu_B), \quad (6)$$

where, for simplicity, the strangeness density is fixed. This gives $T_c = T_c(\mu_B)$. Because from the left and the right of T_c , the pressures are different, we have two baryon densities, for hadrons and for plasma,

$$\rho_{Bh} = \frac{\partial P_h}{\partial \mu_B} = \rho_{Bh}(T, \mu_B), \quad \rho_{Bq} = \frac{\partial P_q}{\partial \mu_B} = \rho_{Bq}(T, \mu_B), \quad (7)$$

which gives two baryon potentials, μ_{Bh} and μ_{Bq} that coincide at the transition temperature:

$$\mu_{Bh}(T_c, \rho_{Bh}) = \mu_{Bq}(T_c, \rho_{Bq}). \quad (8)$$

This defines $T_c = T_c(\rho_{Bh}, \rho_{Bp})$.

The point of a first-order phase transition is the point of instability. Infinitesimally small fluctuations of temperature around T_c will result in finite jumps between two different barion densities in Eq. (7). So that the mixed phase at this point is unstable.

One says that the mixed phase could exist not merely at the transition point, but also in a region around it. This is explained as being due to the Maxwell construction that is demonstrated in Fig. 3 for the pressure as a function of the reduced barion volume

$$v_B \equiv \frac{1}{\rho_B} .$$

Here, the standard behavior of the pressure under a first-order phase transition [56] is corrected by replacing the part, corresponding to unstable and metastable states (shown by the dashed line), by the horizontal solid line between the barion volumes

$$v_{Bh} \equiv \frac{1}{\rho_{Bh}} , \quad v_{Bq} \equiv \frac{1}{\rho_{Bq}} .$$

As a result of this construction, the phase diagram of Fig. 1 transforms into that of Fig. 4. The Maxwell construction for the pressure as a function of temperature is equivalent to the smoothing of the pressure, as is shown in Fig. 5. The mixed phase exists between the low, T_n , and upper, T_n^* , nucleation temperatures.

However, as is evident from Fig. 3, on the coexistence line, one has

$$\frac{\partial P}{\partial v_B} = 0 \quad (v_{Bh} < v_B < v_{Bq}) . \quad (9)$$

This implies that the isothermal compressibility

$$\kappa_T = - \frac{1}{v_B} \left(\frac{\partial P}{\partial v_B} \right)^{-1}$$

is divergent everywhere in the region of the mixed phase existence:

$$\kappa_T \rightarrow \infty \quad (T_n < T < T_n^*) . \quad (10)$$

The divergence of the compressibility means instability, since infinitesimally weak pressure fluctuations would lead to the system explosion during the short explosion time

$$t_{exp} \sim \frac{1}{\kappa_T} .$$

That is, the fireball would explode even before it could equilibrate.

Concluding, if deconfinement would be a first-order phase transition, then, formally, a mixed hadron-quark-gluon phase could arise around the transition point, however such a mixed state is strongly unstable and, in reality, cannot exist as an equilibrium phase. In addition, as QCD lattice simulations prove [11-15], deconfinement is not a first-order transition, but rather a crossover.

3 Mesoscopic mixed state

There can exist another type of mixed state that can be called mesoscopic mixed hadron-quark-gluon matter. The term "mesoscopic" means that the typical size l_g of the arising germs of one phase inside the other is between the mean interparticle distance and the system size:

$$a \ll l_g \ll L . \quad (11)$$

Below the deconfinement temperature, these are the germs of quark-gluon plasma surrounded by hadron matter. And above the transition temperature, these are the germs of hadron matter inside quark-gluon plasma.

For the mesoscopic mixed state, pressure is uniquely defined and does not require the phase transition to be of the first order. Generally, it can be of any order, including the crossover type [19,57]. Mesoscopic mixed state can exist in a large temperature interval between the low and upper nucleation temperatures.

The mesoscopic mixed state is basically different from the macroscopic one, exhibiting the following main features.

(i) The germs of competing phases do not need to be in absolute equilibrium. They can have finite lifetime t_g . But they are to be in quasi-equilibrium, such that the local equilibration time be essentially shorter than their lifetime:

$$t_{loc} \ll t_g . \quad (12)$$

(ii) The spatial distribution of germs at a snapshot is random. They form no ordered spatial structure, such as domains.

(iii) The spatial distribution of germs is also random with respect to repeated experiments.

(iv) The typical size of the germs is mesoscopic in the sense of Eq. (11).

(v) The germ geometry is of multiscale nature. Their shapes are not regular, but are rather ramified. And the sizes l_g lie in a dense interval $[l_g^{min}, l_g^{max}]$, such that

$$a \ll l_g^{min} < l_g^{max} \ll L .$$

The description of the mesoscopic mixed state has to take into account the generic random nature of the spatial germ distribution. The main ideas of the theory are as follows [19,57,58]. We keep in mind a mixture of two phases, e.g., hadron matter and quark-gluon plasma.

At a snapshot, the system volume is divided onto the volumes of different phases,

$$\mathbb{V} = \mathbb{V}_1 \bigcup \mathbb{V}_2 ,$$

separated by the Gibbs equimolecular separating surface, for which extensive observable quantities are additive. This concerns as well the number of particles in each phase and the related volumes:

$$N = N_1 + N_2 , \quad V = V_1 + V_2 , \quad (13)$$

with $V_\nu \equiv \text{mes}\mathbb{V}_\nu$. Mathematically, the separation is characterized by the manifold indicator functions

$$\xi_\nu(\mathbf{r}) = \begin{cases} 1, & \mathbf{r} \in \mathbb{V}_\nu \\ 0, & \mathbf{r} \notin \mathbb{V}_\nu , \end{cases} \quad (14)$$

where \mathbf{r} is a spatial variable and $\nu = 1, 2$ enumerates the phases.

At a snapshot, the mixture needs to be described by a representative statistical ensemble $\{\mathcal{H}, \hat{\rho}(\xi)\}$, where \mathcal{H} is the space of microstates and $\hat{\rho}(\xi)$ is a statistical operator [58]. The space of microstates is given by the fiber space

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 , \quad (15)$$

with the fiber bases \mathcal{H}_ν being weighted Hilbert spaces. The statistical operator is normalized as

$$\text{Tr} \int \hat{\rho}(\xi) \mathcal{D}\xi = 1 , \quad (16)$$

by taking the trace over the quantum degrees of freedom and averaging over the random germ spatial configurations defined through the functional integral over the manifold indicator functions (14).

To construct a representative ensemble, one defines the internal energy

$$E = \text{Tr} \int \hat{\rho}(\xi) \hat{H}(\xi) \mathcal{D}\xi \quad (17)$$

and all constraining quantities

$$C_i = \text{Tr} \int \hat{\rho}(\xi) \hat{C}_i(\xi) \mathcal{D}\xi , \quad (18)$$

required for the unique description of the system. The statistical operator is found from the principle of minimal information, by minimizing the information functional

$$\begin{aligned} I[\hat{\rho}(\xi)] = & \text{Tr} \int \hat{\rho}(\xi) \ln \hat{\rho}(\xi) \mathcal{D}\xi + \\ & + \lambda_0 \left[\text{Tr} \int \hat{\rho}(\xi) \mathcal{D}\xi - 1 \right] + \beta \left[\text{Tr} \int \hat{\rho}(\xi) \hat{H}(\xi) \mathcal{D}\xi - E \right] + \\ & + \sum_i \lambda_i \left[\text{Tr} \int \hat{\rho}(\xi) \hat{C}_i(\xi) \mathcal{D}\xi - C_i \right] , \end{aligned} \quad (19)$$

in which λ_0 , β , and λ_i are Lagrange multipliers.

The minimization yields the statistical operator

$$\hat{\rho}(\xi) = \frac{1}{Z} \exp\{-\beta H(\xi)\} , \quad (20)$$

with the grand Hamiltonian

$$H(\xi) = \hat{H}(\xi) - \sum_i \mu_i \hat{C}_i(\xi) , \quad (21)$$

where $\mu_i \equiv -\lambda_i T$. The partition function is

$$Z = \text{Tr} \int \exp\{-\beta H(\xi)\} \mathcal{D}\xi ,$$

and $\beta = 1/T$ is inverse temperature.

Let us introduce the effective Hamiltonian \tilde{H} defined by the equality

$$\int \exp\{-\beta H(\xi)\} \mathcal{D}\xi = \exp(-\beta \tilde{H}) . \quad (22)$$

After this, the partition function reduces to the form

$$Z = \text{Tr} e^{-\beta \tilde{H}} ,$$

containing only the trace over quantum degrees of freedom.

The geometric weights of each phase are given by the expressions

$$w_\nu = \int \xi_\nu(\mathbf{r}) \mathcal{D}\xi, \quad (23)$$

satisfying the normalization condition

$$w_1 + w_2 = 1 . \quad (24)$$

This provides the minimum for the grand potential

$$\Omega = -T \ln \text{Tr} e^{-\beta \tilde{H}} ,$$

that can be found from the conditions

$$\frac{\partial \Omega}{\partial w_\nu} = 0 \quad \frac{\partial^2 \Omega}{\partial w_\nu^2} > 0 , \quad (25)$$

taking into account normalization (24). The phase weights (23) play the role of additional order parameters characterizing the mixed state [19,57,60,61].

The mesoscopic mixed state is stable, with deconfinement being rather a sharp crossover.

4 Microscopic mixed state

The third type of mixture is termed microscopic because hadrons are uniformly intermixed with quark-gluon plasma, without forming either germs or droplets. Such a mixed state can be treated by the theory of clustering matter [6,7], considering hadrons as quark clusters. Each kind of clusters, enumerated by the index i , is characterized by the baryon number B_i , strangeness S_i , and compositeness z_i . The latter shows the number of quarks forming a cluster of that type. For instance, the quark compositeness is 1, meson compositeness is 2, and the nucleon compositeness is 3.

The space of microstates for the mixture is the tensor product

$$\mathcal{M} = \bigotimes_i \mathcal{F}_i , \quad (26)$$

in which \mathcal{F}_i is the Fock space for the i -clusters.

The density of i -clusters is

$$\rho_i = \zeta_i \int n_i(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^3}, \quad (27)$$

where ζ_i is a degeneracy factor and $n_i(\mathbf{k})$ is a momentum distribution. The total mean quark density is

$$\rho = \sum_i z_i \rho_i. \quad (28)$$

The cluster weights are defined by the ratio

$$w_i \equiv \frac{z_i \rho_i}{\rho}, \quad (29)$$

which gives $w_i = w_i(\rho_B, \rho_S, T)$. By definition, one has

$$0 \leq w_i \leq 1, \quad \sum_i w_i = 1.$$

The Hamiltonian of a microscopic mixture, generally, has the form

$$\hat{H} = \sum_i \hat{H}_i + \frac{1}{2} \sum_{i \neq j} \hat{H}_{ij}, \quad (30)$$

in which the first term is the sum of the channel Hamiltonians and the second term corresponds to cluster interactions. Modeling the Hamiltonian, one often assumes its dependence on density and/or temperature. For example, the effective particle spectra are often defined as functions of temperature [62]. Therefore, in the definition of the grand Hamiltonian,

$$H = \hat{H} - \sum_i \mu_i \hat{N}_i + CV, \quad (31)$$

one has to include the term CV guaranteeing statistical correctness for the approach. To this end, it is necessary to require the validity of the conditions

$$\left\langle \frac{\partial H}{\partial \rho_i} \right\rangle = 0, \quad \left\langle \frac{\partial H}{\partial T} \right\rangle = 0. \quad (32)$$

The latter reduce to the equations

$$\frac{\partial C}{\partial \rho_i} = -\frac{1}{V} \left\langle \frac{\partial H}{\partial \rho_i} \right\rangle, \quad \frac{\partial C}{\partial T} = -\frac{1}{V} \left\langle \frac{\partial H}{\partial T} \right\rangle, \quad (33)$$

defining $C = C(\{\rho_i\}, T)$.

Only under the conditions of the statistical correctness (32), the theory becomes self-consistent and satisfies all thermodynamic relations:

$$\begin{aligned} P &= -\frac{\Omega}{V} = -\frac{\partial \Omega}{\partial V} & E &= \frac{1}{V} \langle \hat{H} \rangle = T \frac{\partial P}{\partial T} - P + \mu_B \rho_B + \mu_S \rho_S, \\ S &= \frac{\partial P}{\partial T} = \frac{1}{T} (E + P - \mu_B \rho_B - \mu_S \rho_S), & \rho_i &= \frac{1}{V} \langle \hat{N}_i \rangle = \frac{\partial P}{\partial \mu_i} \end{aligned}$$

$$\rho_B = \sum_i B_i \rho_i = \frac{\partial P}{\partial \mu_B}, \quad \rho_S = \sum_i S_i \rho_i = \frac{\partial P}{\partial \mu_S}.$$

It is a common mistake, widely spread in literature, when the authors forget about statistical correctness, because of which the obtained results cannot be reliable.

Taking into account cluster interactions may seem to be a problem, since there can exist various quark clusters, whose interactions are not known. This obstacle can be avoided in the following way. Let us consider the reaction of fusion of two clusters, say a cluster a and cluster b , into one cluster i , with all compositeness numbers larger than one, so that there is the conservation of compositeness,

$$z_a + z_b = z_i,$$

and the conservation of mass,

$$m_a + m_b + \Phi_{ab} = m_i,$$

where Φ_{ab} is the interaction energy of two clusters. For the same fusion, in the presence of a third cluster j , the mass conservation reads as

$$m_a + m_b + m_j + \Phi_{ab} + \Phi_{aj} + \Phi_{bj} = m_i + m_j + \Phi_{ij}.$$

From these relations, it follows the *potential scaling law*

$$\frac{\Phi_{ij}}{z_i z_j} = \frac{\Phi_{ab}}{z_a z_b}. \quad (34)$$

This law allows us to express all cluster interactions through one known interaction, e.g., through the nucleon-nucleon interaction,

$$\Phi_{ij} = \frac{z_i z_j}{9} \Phi_{NN}, \quad (35)$$

which is well known [49].

The microscopic hadron-quark-gluon mixture is stable, with deconfinement being a sharp crossover [6,7], in good agreement with the QCD lattice simulations [11-15]. In the case of a crossover, the deconfinement temperature can be defined as the point where the derivatives of observables have a maximum, which gives about 170 MeV. Of course, considering different observables can result in slightly different deconfinement temperatures, which is the common situation for crossovers, where the crossover temperature is defined conditionally. Numerical simulations [63,64] show that pion clusters survive till around $2T_c$.

5 Static and dynamic stability

One more problem that arises in considering the coexistence of clusters of different types is the possibility of their spatial stratification, when the clusters, previously uniformly mixed, separate in space into domains containing only one kind of clusters. Below, we illustrate this problem by considering a two-component mixture of clusters.

Let the total number of clusters be $N = N_1 + N_2$, existing in the volume $V = V_1 + V_2$. The system can form two kinds of mixture. One situation corresponds to a microscopic mixture, with all clusters being uniformly intermixed in the space. And the other case is when the clusters of each type are spatially separated into different domains, thus forming a macroscopic mixed state. The microscopic mixture is more thermodynamically stable when its free energy F_{mix} is lower than the free energy F_{sep} of the separated state of the macroscopic mixture,

$$F_{mix} < F_{sep} . \quad (36)$$

Calculating the free energy in the correlated mean-field approximation, we use the notation for the mean interaction intensity

$$\Phi_{ij} = \int V_{ij}(\mathbf{r}) g_{ij}(\mathbf{r}) d\mathbf{r} , \quad (37)$$

in which $V_{ij}(\mathbf{r})$ is a vacuum cluster interaction and $g_{ij}(\mathbf{r})$ is the pair correlation function. Then from Eq. (36), we find the condition for the stability of the microscopic mixture

$$\Phi_{12} < \sqrt{\Phi_{11}\Phi_{22}} + \frac{TV}{N_1 N_2} \Delta S_{mix} , \quad (38)$$

where ΔS_{mix} is the entropy of mixing, which can be written as

$$\Delta S_{mix} = -N_1 \ln \frac{N_1}{N} - N_2 \ln \frac{N_2}{N} . \quad (39)$$

In the case of validity of the potential scaling (34), the stability condition (38) reduces to the trivial requirement that the entropy of mixing (39) be positive, which is certainly true. Hence, under the validity of the potential scaling, the microscopic mixture is always more stable and there is no stratification.

The stability condition (38) is derived for an equilibrium situation by comparing the thermodynamic potentials of the microscopic mixture and the separated stratified state. In that sense, it is a static stability condition. But there is also a dynamic stability condition requiring that the spectrum of elementary excitations be positive [65]. Analyzing the dynamic stability, we take into account that the components can move with respect to each other with the velocities \mathbf{v}_1 and \mathbf{v}_2 . Such a relative motion can be due to the fact that the fireball has been formed as a result of two colliding heavy ions or nuclei.

Studying the spectrum of collective excitations of a microscopic mixture in the random-phase approximation, we find that the spectrum is positive, provided that the relative velocity $\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$ does not exceed by the magnitude $v \equiv |\mathbf{v}|$ the critical value

$$v_c = \sqrt{\frac{\rho_2}{m_2 \Phi_{11}} (\Phi_{11}\Phi_{22} - \Phi_{12}^2)} . \quad (40)$$

If $v < v_c$, the microscopic mixture is stable. But if $v > v_c$, the mixture stratifies into macroscopic domains containing different sorts of clusters each. The dynamic instability, leading to the stratification, caused by the mutual motion of components, is called the *counterflow instability*.

In conclusion, we have explained that there are three types of mixed systems, macroscopic, mesoscopic, and microscopic. Each kind of these mixed states is very different

from others, enjoying quite different physical properties and needing principally different theoretical description.

If deconfinement would be a first-order phase transition, there could arise the macroscopic mixed state, where hadron and quark-gluon phases would be located in separate macroscopic spatial domains. However, such a state is not stable and would disappear even before a fireball would equilibrate. In addition, lattice QCD simulations demonstrate that deconfinement is not a first-order transition, but a crossover. Hence, the macroscopic mixed state has no chance to exist. Therefore the naive picture, when one compares the mixed hadron-quark-gluon phase with a boiling water containing gas bubbles, has nothing to do with QCD. Theoretical predictions, based on the macroscopic mixed model, cannot be confronted with experiment.

The real quark-hadron mixed state can be either mesoscopic or microscopic. These states can be stable, with deconfinement being rather a sharp crossover.

Studying a multicomponent mixture, it is necessary to check it with respect to the stratification instability. The components, moving through each other, can also exhibit the counterflow instability. All these effects need to be carefully analyzed before comparing theoretical predictions with experimental observations.

References

- [1] E.V. Shuryak, Phys. Rep. **61**, 71 (1980).
- [2] H. Satz, Phys. Rep. **88**, 349 (1982).
- [3] R. Hagedorn, Riv. Nuovo Cimento **6**, 1 (1983).
- [4] J. Cleymans, R. Gavai, and E. Suhonen, Phys. Rep. **130**, 217 (1986).
- [5] H. Reeves, Phys. Rep. **201**, 335 (1991).
- [6] V.I. Yukalov and E.P. Yukalova, Physica A **243**, 382 (1997).
- [7] V.I. Yukalov and E.P. Yukalova, Phys. Part. Nucl. **28**, 37 (1997).
- [8] H. Satz, Nucl. Phys. A **681**, 3 (2001).
- [9] V.I. Yukalov and E.P. Yukalova, in *Relativistic Nuclear Physics and Quantum Chromodynamics*, edited by A.M. Baldin, V.V. Burov, and A.I. Malakhov (JINR, Dubna, 2001), Vol. 1. p. 109.
- [10] H.B. Meyer, Proc. Sci. (QNP) 009 (2012).
- [11] Y. Aoki, G. Endrodi, Z. Fodor, S. Katz, and K. Szabo, Nature **443** 675 (2006).
- [12] F. Karsch, Prog. Part. Nucl. Phys. **62**, 503 (2009).
- [13] S. Borsanyi, Z. Fodor, C. Hoelbling, S.D. Katz, S. Krieg, C. Ratti, and K. Szabo, J. High Energy Phys. **09**, 073 (2010).
- [14] A. Bazavov et al., Phys. Rev. D **85**, 054503 (2012).
- [15] O. Philipsen, arXiv:1207.5999 (2012).
- [16] R. Stock, Phys. Rep. **135**, 260 (1986).
- [17] R. Clare and D. Strottman, Phys. Rep. **141**, 177 (1986).
- [18] L. McLerran, Rev. Mod. Phys. **58**, 1021 (1986).
- [19] V.I. Yukalov, Phys. Rep. **208**, 395 (1991).
- [20] A.M. Baldin, A.S. Shumovsky, and V.I. Yukalov, Commun. JINR P2-85-830 (1985).
- [21] A.M. Baldin, A.S. Shumovsky, and V.I. Yukalov, Phys. Many-Part. Syst. **10**, 10 (1986).
- [22] D.I. Blokhintsev, J. Exp. Theor. Phys. **6**, 995 (1957).
- [23] A.M. Baldin, Phys. Part. Nucl. **8**, 429 (1977).
- [24] A.V. Efremov, Phys. Part. Nucl. **13**, 613 (1982).

- [25] A.P. Kobushkin and V.P. Shelest, Phys. Part. Nucl. **14**, 1146 (1983).
- [26] M.M. Makarov, Phys. Part. Nucl. **15**, 941 (1984).
- [27] V.V. Burov, V.K. Lukyanov, and A.I. Titov, Phys. Part. Nucl. **15**, 1249 (1984).
- [28] A.M. Baldin, R.G. Nazmitdinov, A.V. Chizhov, A.S. Shumovsky, and V.I. Yukalov, Dokl. Phys. **29**, 952 (1984).
- [29] A.M. Baldin, R.G. Nazmitdinov, A.V. Chizhov, A.S. Shumovsky, and V.I. Yukalov, in *Multiquark Interactions and Quantum Chromodynamics*, edited by A.M. Baldin (JINR, Dubna, 1984), p. 531.
- [30] C.W. Wong, Phys. Rep. **136**, 1 (1986).
- [31] J. Rafelski, Phys. Lett. B **207**, 371 (1988).
- [32] J. Rafelski, Phys. Rep. **88**, 331 (1982).
- [33] L.P. Csernai and J.I. Kapusta, Phys. Rep. **131**, 223 (1986).
- [34] H. Stöcker and W. Greiner, Phys. Rep. **137**, 277 (1986).
- [35] B. Friman, K. Kajantie, and P. Ruuskanen, Nucl. Phys. B **266**, 468 (1986).
- [36] M.I. Polikarpov, Phys. Lett. B **236**, 61 (1990).
- [37] A.P. Vischer, P.J. Siemens, and A.J. Sierk, Z. Phys. A **340**, 315 (1991).
- [38] A.K. Bukenov, A.I. Veselov, and M.I. Polikarpov, Phys. At. Nucl. **55**, 226 (1992).
- [39] H. Heiselberg, C. Pethick, and E. Staubo, Phys. Rev. Lett. **70**, 1355 (1993).
- [40] M. Oleszczuk and J. Polonyi, Ann. Phys. (N.Y.) **227**, 76 (1993).
- [41] A.L. Goodman, J.I. Kapusta, and A.Z. Mekjian, Phys. Rev. C **30**, 851 (1984).
- [42] G. Peilert, J. Randrup, H. Stöcker, and W. Greiner, Phys. Lett. B **260**, 271 (1991).
- [43] A.D. Linde, Rep. Prog. Phys. **47**, 925 (1984).
- [44] A.D. Linde, Phys. Usp. **144**, 177 (1984).
- [45] A. Linde, D. Linde, A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994).
- [46] A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D **69**, 074029 (2004).
- [47] R. Jackiw, Rev. Mod. Phys. **49**, 681 (1977).
- [48] B.I. Birbrair, L.P. Lapina, and V.A. Sadovnikova, Phys. At. Nucl. **24**, 491 (1976).
- [49] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).

- [50] S. Mukherjee, R. Nag, S. Sanyal, T. Morii, J. Morishita, and M. Tsuge, Phys. Rep. **231**, 201 (1993).
- [51] V.I. Yukalov and E.P. Yukalova, in *Relativistic Nuclear Physics and Quantum Chromodynamics*, edited by A.M. Baldin and V.V. Burov (JINR, Dubna, 2000), Vol. 2, p. 238.
- [52] J. Letessier and J. Rafelski, Phys. Rev. C **67**, 031902 (2003).
- [53] U. Kraemmer and A. Rebhan, Rep. Prog. Phys. **67**, 351 (2004).
- [54] D.J. Gross, R.D. Pisarski, and L.G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).
- [55] A.I. Vainshtein V.I. Zakharov, V.A. Novikov, and M.A. Shifman, Phys. Usp. **136**, 553 (1982).
- [56] D. ter Haar, *Lectures on Selected Topics in Statistical Mechanics* (Pergamon, Oxford, 1977).
- [57] V.I. Yukalov, Int. J. Mod. Phys. B **17**, 2333 (2003).
- [58] V.I. Yukalov, Symmetry **2**, 40 (2010).
- [59] V.I. Yukalov, Int. J. Mod. Phys. B **21**, 69 (2007).
- [60] V.I. Yukalov, in *Problems of Statistical Mechanics*, edited by N.N. Bogolubov (JINR, Dubna, 1978), p.437.
- [61] V.I. Yukalov, Phys. Lett. A **81**, 433 (1981).
- [62] C. Chen, C. De Tar, and T. De Grand, Phys. Rev. D **37**, 247 (1988).
- [63] V.S. Filinov, Y.B. Ivanov, M. Bonitz, P.R. Levashov, and V.E. Fortov, Part. Nucl. Lett. **8**, 823 (2011).
- [64] C. Ratti, R. Bellwied, M. Cristoforetti, and M. Barbaro, Phys. Rev. D **85**, 014004 (2012).
- [65] V.I. Yukalov and E.P. Yukalova, Laser Phys. Lett. **1**, 50 (2004).

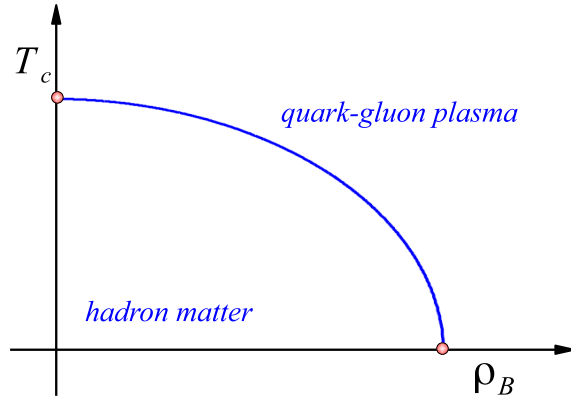


Figure 1: First-order phase-transition temperature T_c as a function of barion density.

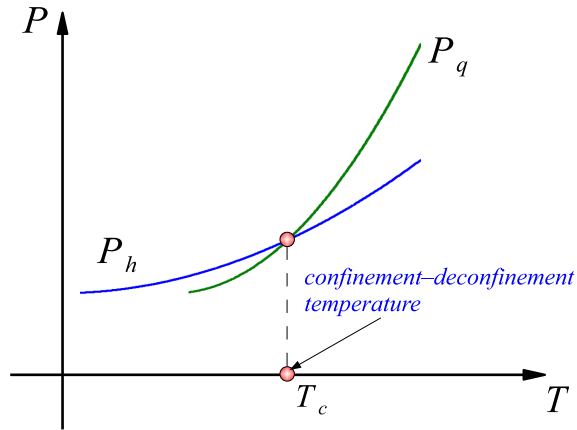


Figure 2: Pressures of the hadron matter and quark-gluon phase as functions of temperature.

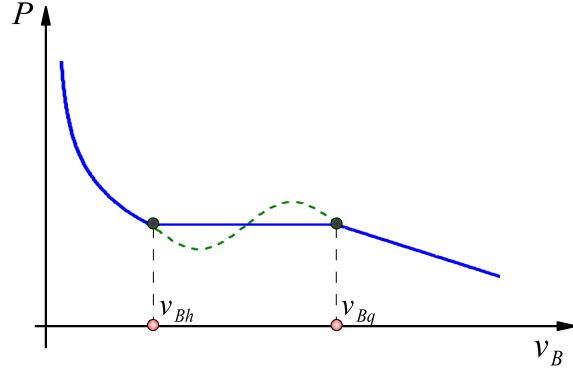


Figure 3: Maxwell construction for the pressure as a function of the reduced barion volume.

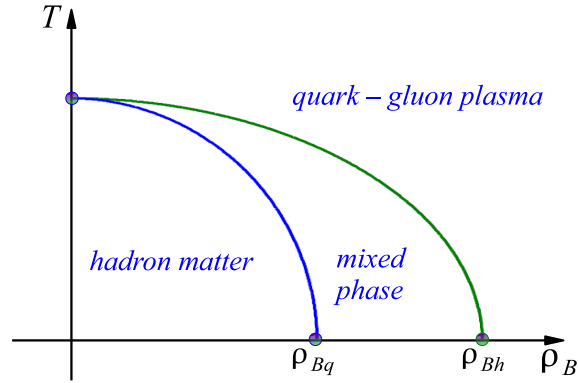


Figure 4: Mixed hadron-quark-gluon phase formally appearing around a first-order phase transition as a result of the Maxwell construction.

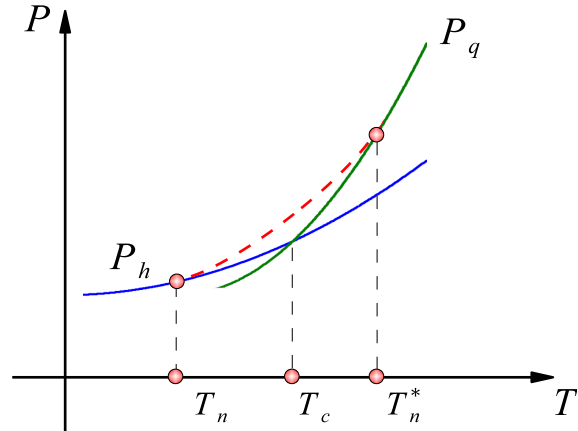


Figure 5: Smoothed pressure (dashed line) as a function of temperature, corresponding to the Maxwell construction.